

## Indeterminate Forms:

- $\frac{0}{0}$
- $\frac{\pm\infty}{\pm\infty}$
- $\infty - \infty$  (or  $-\infty + \infty$ )
- $0 \cdot \infty$
- $0^0$
- $1^\infty$
- $\infty^0$

Determinate Forms: (where  $a, b, c$  are arbitrary fixed real numbers, and we assume  $a \neq 0, c \neq 1$ )

- $\frac{a}{0}$
- $\frac{b}{\pm\infty}$  or  $\frac{\pm\infty}{b}$
- $\infty + \infty$  (or  $-\infty + -\infty$ )
- $a \cdot \infty$
- $\pm\infty \cdot \pm\infty$
- $0^\infty$
- $c^\infty$
- $\infty^a$  or  $\infty^\infty$

For each of the problems 1-10, evaluate the limit or conclude that the function tends to  $\infty, +\infty$ , or  $-\infty$ .

1.  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 3}{x^3 + 4}$
2.  $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 4}{x^2 - 2x + 3}$
3.  $\lim_{x \rightarrow \infty} \frac{x^4 - 2x^2 + 6}{x^2 + 7}$
4.  $\lim_{x \rightarrow +\infty} [x - \sqrt{x^2 - a^2}]$  where  $a$  is an arbitrary fixed real number
5.  $\lim_{x \rightarrow -\infty} [x - \sqrt{x^2 - a^2}]$  where  $a$  is an arbitrary fixed real number
6.  $\lim_{x \rightarrow 1^+} \frac{x-1}{\sqrt{x^2-1}}$
7.  $\lim_{x \rightarrow 1^-} \frac{x-1}{\sqrt{x^2-1}}$
8.  $\lim_{x \rightarrow +\infty} \frac{x^2+1}{x^{\frac{3}{2}}}$
9.  $\lim_{x \rightarrow 2^-} \frac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$
10.  $\lim_{x \rightarrow +\infty} [\sqrt{x^2 + 2x} - x]$
11. Suppose  $f(x) \rightarrow +\infty$  and  $g(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$ . Find examples of functions  $f$  and  $g$  with these properties and such that:
  - a.  $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = +\infty$
  - b.  $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = -\infty$
  - c.  $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = A$ , where  $A$  is an arbitrary real number

12. Suppose  $f(x) \rightarrow \pm\infty$  and  $g(x) \rightarrow \pm\infty$  as  $x \rightarrow +\infty$ . Find examples of functions  $f$  and  $g$  with these properties and such that:

a.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = +\infty$

b.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = -\infty$

c.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = A$ , where  $A$  is an arbitrary real number

13. Suppose  $f(x) \rightarrow 0$  and  $g(x) \rightarrow 0$  as  $x \rightarrow +\infty$ . Find examples of functions  $f$  and  $g$  with these properties and such that:

a.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = +\infty$

b.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = -\infty$

c.  $\lim_{x \rightarrow +\infty} \left[ \frac{f(x)}{g(x)} \right] = A$ , where  $A$  is an arbitrary real number

14. Suppose  $f(x) \rightarrow 0$  and  $g(x) \rightarrow \pm\infty$  as  $x \rightarrow +\infty$ . Find examples of functions  $f$  and  $g$  with these properties and such that:

a.  $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = +\infty$

b.  $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = -\infty$

c.  $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = A$ , where  $A$  is an arbitrary real number

**Extra Credit:** Suppose  $f(x) \rightarrow 0$  and  $g(x) \rightarrow 0$  as  $x \rightarrow 0^+$ . Find examples of functions  $f$  and  $g$  with these properties and such that:

a.  $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = 0$

b.  $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = 1$

c.  $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = A$ , where  $A$  is an arbitrary real number and  $A \neq 0$  or  $1$